**Assignment 3**

**Submitted by: Tony Trotter**

**1. Part 1: Forwards**

Consider the following zero-coupon curve. These are spot rates.

Table

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1. Calculate the following forward rates from the spot curve given above, f(0,0.5,1), f(0,1,1.5), f(0,1.5,2), and f(0,0.5,2)

2. Suppose that an investment bank is currently quoting the rate on an FRA off f\*(0,1,1.5) = 4.5% (this should be different than what you found above). Use this f \*(0,1,1.5) = 4.5% and the given zero-coupon curve, construct an arbitrage trading strategy to take advantage of the miss pricing. (Note: it may be helpful to calculate the price of the 1-year and 1.5-year zero-coupon bonds off the curve. You can then setup a synthetic forward contract.)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  | **Trade** | | **Cash Flow** | | |  |
|  |  |  | t=0 | t=1 | t=1.5 |  |
|  |  |  |  |  |  |  |
|  | **Forward Contract** |  |  |  |  |  |
|  | Lender |  | $0.00 | ($100.00) | $102.25 |  |
|  |  |  |  |  |  |  |
|  | **Bond Portfolio** |  |  |  |  |  |
|  | Long 1yr bond |  | **($97.55)** | $100.00 |  |  |
|  |  |  |  |  |  |  |
|  | Short 1.5yr bond |  | $97.55 |  | ($102.00) |  |
|  |  |  |  |  |  |  |
|  | Total CF |  | $0.00 | $0.00 | **$0.2481** |  |
|  |  |  |  |  |  |  |

3. Using the forward rates that you calculated in question #1, suppose that you entered into a forward rate agreement to lend $100 at t = 0.5 to t = 2. Find the value of the forward rate agreement if all interest rates decline by 1 percentage point immediately after you enter the contract. Find the value of the forward rate agreement if all interest rates increase by 1 percentage point immediately after you enter the contract. Hint: do not overcomplicate things. Write down the cash flows to the FRA that you entered. Then calculate the NPV using the new discount rates.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  | **Trade** | | | **Cash Flow** | | | |  |  |
|  |  |  | NPV | t=0 | t=.5 | t=1 | t=1.5 | t=2 |  |
|  |  |  |  |  |  |  |  |  |  |
|  | **Long FRA** | |  |  |  |  |  |  |  |
|  | Lender @ 4.67% | |  |  | ($100.00) | $0.00 | $0.00 | $107.17 |  |
|  |  |  |  |  |  |  |  |  |  |
|  | NPV @ Spot | | ($0.00) |  | ($99.01) | $0.00 | $0.00 | $99.01 |  |
|  | IRR @ Spot | | 0.00% |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | NPV @ Spot+1% | | ($1.43) |  | ($98.52) | $0.00 | $0.00 | $97.09 |  |
|  | IRR @ Spot+1% | | -0.49% |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | NPV @ Spot -1% | | $1.47 |  | ($99.50) | $0.00 | $0.00 | $100.97 |  |
|  | IRR @ Spot+1% | | 0.49% |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

4. What does your answer to #3 imply about (i) the modified duration of cash inflows from the FRA vs. (ii) the modified duration of cash outflows from the FRA.

Provide calculations here to support your conclusion

Modified duration measures the average cash-weighted [term to maturity](https://www.investopedia.com/terms/t/termtomaturity.asp) of a bond. Additionally, all other risk factors equal, bonds with a higher duration have greater price [volatility](https://www.investopedia.com/terms/v/volatility.asp) than bonds with a lower duration. The cash inflows occur further into the future than the cash outflows. This implies that the inflows will have a higher modified duration and a higher price sensitivity to interest rate movements than the outflows.

The Macaulay Duration of a zero-coupon bond is its time to maturity; the MD of the FRA’s from question #3 are as follows:

**Cash Outflows**

**Cash Inflows**

After careful analysis, the key findings regarding the MDs of the FRAs from question #3 became clear. They are listed below.

**Key Findings:**

* *As the discount rate (increases)decreases, the modified duration of a forward agreement goes (down)up.*
* *As time to maturity (increases)decreases, the modified duration of a forward rate agreement goes (up)down.*

**2. Part 2: Interest Rate Swaps**

Use the following term structure of zero-coupon yields to answer the questions below.

Table

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1. Find the fair fixed rate for a 2-year interest rate swap. The swap terms provide for the exchange of fixed for floating rate payments every six-months where floating rates payments are equal to six-month LIBOR. Note: In the lecture, we computed the fair fixed rate for a swap with quarterly payments. This is slightly different

2. For this question, work with the fixed and floating leg of the swap separately. Calculate the modified duration of the floating leg. It may be helpful to work with the floating leg as if it were a floating rate bond Calculate the modified duration of the fixed leg. It may be helpful to work with the fixed leg as if it were a fixed rate coupon bond. Briefly describe how to compute the modified duration of the swap contract

r = .0396

coupon = 1.98

FV = 100

r = .0406

r = .0386

The modified duration of a swap is a measure of the swap’s value sensitivity to interest rate changes. Therefore, at inception, the duration of a swap is equal to the difference between the durations of the two legs of the swap. This is because payments on the fixed leg of an interest rate swap are equivalent to those of a [fixed-rate bond](https://investment-and-finance.net/finance/f/fixed-rate-bond.html), and payments on the floating leg are comparable to those of a [floating-rate bond](https://investment-and-finance.net/derivatives/s/floating-rate-bond.html).

It then follows that the net settlement of the cash flows of the swap can be used to figure out the swap's duration. Additionally, both fixed and floating-rate bonds should be priced at par value at the outset because the initial value of the swap is zero. After inception, the modified duration should be calculated as the weighted average of the MD’s of the two legs of the swap.

3. Suppose that exactly one half of a year has passed, and the current term structure of zero-coupon yields is shown below:

A picture containing shape

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Suppose you entered the fixed-for-floating rates swap (in Question 1) one-half year ago and specified a notional value of $100. What is the value of your position today?

Have you made money or lost money, and why? (Think about how sensitive each of the swap legs is to changes in interest rates - this may help

If I had entered the swap from the position of receiving fixed (floating) payments and making floating (fixed) payments the value of my contract would have gone down (up) by $1.49.

The fixed rate leg of the swap is vulnerable to increases in interest rates because, the coupon payments are fixed but are discounted at a higher rate. Conversely, the floating rate leg of the swap has its coupons discounted at higher rates in the event of a parallel increase in interest rates, but its coupons also rise which offsets the impact of the additional discounting.